

K-meson neutrinoless double muon decay as a probe of neutrino masses and mixings

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Abstract

Recently an upper bound on the rate of the lepton number violating decay $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ has been significantly reduced by the E865 experiment at BNL and further improvement is expected in the near future. We study this process as a possible source of information on neutrino masses and mixings. We find that it is insensitive to the light(eV domain) and heavy(GeV domain) neutrinos. However due to the effect of a resonant enhancement this decay is very sensitive to neutrinos ν_j in the mass region $245 \text{ MeV} \leq m_{\nu_j} \leq 389 \text{ MeV}$. At present experimental sensitivity we deduce new stringent limits on the neutrino mixing matrix element $U_{\mu j}$ for neutrino masses in this region.

1 Introduction

Nowadays there are sufficient indications to believe that neutrinos are massive particles mixing with each other [1]. These indications come from both experimental and theoretical sides. The solar neutrinos deficit, the atmospheric neutrino anomaly and the results of the LSND neutrino oscillation experiment, all can be explained in terms of neutrino oscillations implying non-zero neutrino masses and mixings. On the theoretical side almost all phenomenologically viable models of the physics beyond the standard model(SM) predict non-zero masses for neutrinos which can be either Majorana or Dirac particles.

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Majorana masses violate the conservation of total lepton number by two units $\Delta L = 2$. Therefore lepton number violating (\mathbb{L}) processes represent a most appropriate tool to address the question of whether neutrinos are Majorana or Dirac particles. Various \mathbb{L} processes have been studied in the literature in this respect. Among them there are the neutrinoless nuclear double beta ($0\nu\beta\beta$) decay [2, 3], the decay $K^+ \rightarrow \mu^+\mu^+\pi^-$ [4, 5, 6, 7, 8], nuclear muon to positron [9] or to antimuon [10] conversion, trimuon production in neutrino-nucleon scattering [11], the process $e^+p \rightarrow \bar{\nu}l_1^+l_2^+X$, relevant for HERA [12], as well as direct production of heavy Majorana neutrinos at various colliders [13]. The analysis made in the literature [6, 14] leads to the conclusion that if these processes are mediated by the Majorana neutrino exchange then, except $0\nu\beta\beta$ -decay, they can hardly be observed experimentally. This analysis relies on the current neutrino oscillation data, and on certain assumptions about the neutrino mass matrix. In the present paper we concentrate on the neutrinoless double muon decay of kaon $K^+ \rightarrow \mu^+\mu^+\pi^-$. We will show that despite the above conclusion being true for contributions of the neutrino states much lighter or much heavier than the typical energy of the $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay, there is still a special window in the neutrino sector which can be efficiently probed by searching for this process. This window is in the neutrino mass range $245 \text{ MeV} \leq m_{\nu_j} \leq 389 \text{ MeV}$, where the s-channel neutrino contribution to the $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay is resonantly enhanced, therefore making this decay very sensitive to the neutrinos in this mass domain. If neutrinos with masses in this region exist, then from present experimental data we can extract stringent limits on their mixing with ν_μ . We derive these limits from the upper bound on the branching ratio of $K^+ \rightarrow \mu^+\mu^+\pi^-$ recently obtained by E865 experiment at BNL [15].

2 $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay in Standard Model with Majorana neutrinos

In the SM extension with Majorana neutrinos there are two lowest order diagrams, shown in Fig.1, which contribute to $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay. These diagrams were first considered long ago in Refs. [4, 5]. Here we are studying previously overlooked aspects of this decay. We concentrate on the s-channel neutrino exchange diagram in Fig. 1(a) which plays a central role in our analysis. The t-channel diagram in Fig. 1(b) requires in general a detailed hadronic structure calculation. In Ref. [5] this diagram was evaluated in the Bethe-Salpeter approach and shown to be an order of magnitude smaller than the diagram in Fig. 1(a), for light and

intermediate mass neutrinos. As we will see, in the neutrino mass domain of our main interest, the diagram in Fig. 1(a) absolutely dominates over the t-channel diagram in Fig. 1(b), independently of hadronic structure.

The contribution from the factorizable s-channel diagram in Fig. 1(a) can be calculated in a straightforward way, without referring to any hadronic structure model. A final result for the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay rate is given by

$$\begin{aligned} \Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-) = & c \int_{s_1^-}^{s_1^+} ds_1 \left| \sum_k \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \right|^2 G\left(\frac{s_1}{m_K^2}\right) + \\ & 2 \frac{c}{m_K^2} \text{Re} \sum_{k,n} \left[\int_{s_1^-}^{s_1^+} ds_1 \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \int_{s_2^-}^{s_2^+} ds_2 \left(\frac{U_{\mu n}^2 m_{\nu n}}{s_2 - m_{\nu n}^2} \right)^* H\left(\frac{s_1}{m_K^2}, \frac{s_2}{m_K^2}\right) \right]. \end{aligned} \quad (1)$$

The unitary mixing matrix U_{ij} relates $\nu'_i = U_{ij} \nu_j$ weak ν' and mass ν neutrino eigenstates. The numerical constant in Eq. (1) is $c = (G_F^4/32)(\pi)^{-3} f_\pi^2 f_K^2 m_K^5 |V_{ud}|^2 |V_{us}|^2$, where $f_K = 1.28 f_\pi$, $f_\pi = 0.668 m_\pi$ and $m_K = 494$ MeV is the K-meson mass. The functions $G(z)$ and $H(z_1, z_2)$ in Eq. (1) after the phase space integration can be written in an explicit algebraic form

$$\begin{aligned} G(z) = & \frac{\phi(z)}{z^2} [h_{+-}(z)h_{--}(z) - x_\pi^2 h_{-+}(z)] [x_\mu^2 + z - (x_\mu^2 - z)^2] \\ H(z_1, z_2) = & h_{--}(z_1)h_{--}(z_2) + x_\pi^2 [r_+(z_1 z_2) - x_\mu^2 t(z_1, z_2, 1)] - r_-(z_1 z_2) t(z_1, z_2, x_\mu). \end{aligned} \quad (2)$$

Here we defined $x_i = m_i/m_K$ and $h_{\pm\pm}(z) = z \pm x_\pi^2 \pm x_\mu^2$, $r_\pm(z_1 z_2) = z_1 z_2 - x_\pi^2 \pm x_\mu^4$, $t(z_1, z_2, z_3) = z_1 + z_2 - 2z_3^2$, $\phi(z) = \lambda^{1/2}(1, x_\mu^2, z) \lambda^{1/2}(z, x_\mu^2, x_\pi^2)$ with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$. The integration limits in Eq. (1) are

$$\begin{aligned} s_1^- = & m_K^2 (x_\pi + x_\mu)^2, \quad s_1^+ = m_K^2 (1 - x_\mu)^2, \\ s_2^\pm = & \frac{m_K^2}{2y} [2y(1 + x_\mu^2) - (1 + y - x_\mu^2)h_{-+}(y) \pm \phi(y)] \end{aligned} \quad (3)$$

with $y = s_1/m_K^2$.

First we assume that neutrinos can be separated into light ν_k and heavy N_k states, with masses $m_{\nu i} \ll \sqrt{s_1^-}$ and $\sqrt{s_1^+} \ll M_{Nk}$. Then the Eq. (1) can be approximately written as

$$\begin{aligned} \Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-) = & |\langle m_\nu \rangle_{\mu\mu}|^2 m_K^{-1} \mathcal{A}_\nu + \left| \langle M_N^{-1} \rangle_{\mu\mu} \right|^2 m_K^3 \mathcal{A}_N \\ & - 2 \text{Re} [\langle m_\nu \rangle_{\mu\mu} \langle M_N^{-1} \rangle_{\mu\mu}] m_K \mathcal{A}_{\nu N}, \end{aligned} \quad (4)$$

where the average neutrino masses are determined in the standard way

$$\langle m_\nu \rangle_{\mu\mu} = \sum_{k=light} U_{\mu k}^2 m_{\nu k}, \quad \langle M_N^{-1} \rangle_{\mu\mu} = \sum_{k=heavy} U_{\mu k}^2 M_{Nk}^{-1}. \quad (5)$$

The approximate formula (4) can be used for extracting limits on the average neutrino masses from the experimental data. This leads to the limits: $|\langle m_\nu \rangle| \leq Exp(\nu)$, $|\langle M_N^{-1} \rangle| \leq Exp(N)$. We point out that in this case the upper bounds must satisfy the consistency conditions $Exp(\nu) < \sqrt{s_1^-} \sim m_K$, $Exp(N)^{-1} > \sqrt{s_1^+} \sim m_K$. Otherwise the so derived limits are not applicable to $\langle m_\nu \rangle$, $\langle M_N^{-1} \rangle$ as it is in Refs. [7, 8, 12]. If consistency conditions are not satisfied one has to use the initial formula (1). Similar consistency conditions take place for the other \mathbb{L} processes.

The dimensionless coefficients in Eq. (4) are $\mathcal{A}_\nu = 4.0 \times 10^{-31}$, $\mathcal{A}_N = 7.0 \times 10^{-32}$, $\mathcal{A}_{\nu N} = 1.7 \times 10^{-31}$. With these numbers we can estimate the current upper bound on the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay rate from the experimental data on other processes.

Atmospheric and solar neutrino oscillation data, combined with the tritium beta decay endpoint, allow one to set upper bounds on the masses of the known three neutrinos [16] $m_{e,\mu,\tau} \leq 3$ eV. Thus in the three neutrino scenario one gets $|\langle m_\nu \rangle_{\mu\mu}| \leq 9$ eV. In this case we derive from Eq. (4) the following branching ratio

$$\mathcal{R}_{\mu\mu} = \frac{\Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-)}{\Gamma(K^+ \rightarrow all)} \leq 3.0 \times 10^{-30} \quad (3 \text{ light neutrino scenario}). \quad (6)$$

Assuming the existence of heavy (*i.e.* order \sim GeV) mass neutrinos N, we may obtain an upper bound on $\Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-)$ using the current LEP limit on heavy stable neutral leptons $M_N \geq 39.5$ GeV [17], which leads to $|\langle M_N^{-1} \rangle_{\mu\mu}| \leq n(39.5 \text{ GeV})^{-1}$, where n is the number of heavy neutrinos. This limit being substituted in Eq. (4) results in the upper bound

$$\mathcal{R}_{\mu\mu} \leq 2.0 \times 10^{-19} \quad (3 \text{ light} + 1 \text{ heavy neutrino scenario}). \quad (7)$$

Direct searches for $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay by E865 experiment at BNL [15] give

$$\mathcal{R}_{\mu\mu} \leq 3.0 \times 10^{-9} \quad (90\%CL, \text{ Ref. [15]}) \quad (8)$$

Comparison of this experimental bound with the theoretical predictions in Eqs. (6), (7) clearly shows that both cases are far from being ever detected. On

the other hand experimental observation of $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay at larger rates would indicate some new physics beyond the SM, or, as we will see, the presence of an extra neutrino state ν_j with the mass in the hundred MeV domain. Let us note that such neutrinos are not excluded by the LEP neutrino counting experiments measuring the Z-boson invisible width. These experiments set limits not on the number of light massive neutrinos but on the number of active neutrino flavors $N_\nu = 3$. Thus extra massive neutrino states ν_j can appear as a result of mixing of the three active neutrinos with certain number of sterile neutrinos. These massive neutrinos are searched for in many experiments [18]. The ν_j states would manifest themselves as peaks in differential rates of various processes, and can give rise to significant enhancement of the total rate if their masses lie in an appropriate region.

3 Hundred-MeV neutrinos in $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ -decay. Resonant case.

Assume there exists a massive Majorana neutrino ν_j with the mass m_j in the range

$$\sqrt{s_1^-} \approx 245 \text{ MeV} \leq m_j \leq \sqrt{s_1^+} \approx 389 \text{ MeV}. \quad (9)$$

Then the s-channel diagram in Fig. 1(a) blows up because the integrand of the first term in Eq. (1) has a non-integrable singularity at $s = m_j^2$. Therefore, in this resonant domain the total ν_j -neutrino decay width Γ_{ν_j} has to be taken into account. This can be done by the substitution $m_j \rightarrow m_j - (i/2)\Gamma_{\nu_j}$. The total decay width Γ_{ν_j} of the Majorana neutrino ν_j with the mass in the resonant domain (9) receives contributions from the following decay modes:

$$\nu_j \longrightarrow \begin{cases} e^+ \pi^-, e^- \pi^+, \mu^+ \pi^-, \mu^- \pi^+, \\ e^+ e^- \nu_e^c, e^+ \mu^- \nu_\mu^c, \mu^+ e^- \nu_e^c, \mu^+ \mu^- \nu_\mu^c, \\ e^- e^+ \nu_e, e^- \mu^+ \nu_\mu, \mu^- e^+ \nu_e, \mu^- \mu^+ \nu_\mu. \end{cases} \quad (10)$$

Since $\nu_j \equiv \nu_j^c$ it can decay in both $\nu_j \rightarrow l^- X (\Delta L = 0)$ and $\nu_j \rightarrow l^+ X^c (\Delta L = 2)$ channels. Calculating partial decay rates we obtain

$$\Gamma(\nu_j \rightarrow l \pi) = |U_{lj}|^2 \frac{G_F^2}{4\pi} f_\pi^2 m_j^3 F(y_l, y_\pi) \equiv |U_{lj}|^2 \Gamma_2^{(l)}, \quad (11)$$

$$\Gamma(\nu_j \rightarrow l_1 l_2 \nu) = |U_{l_1 j}|^2 \frac{G_F^2}{192\pi^3} m_j^5 H(y_{l_1}, y_{l_2}) \equiv |U_{l_1 j}|^2 \Gamma_3^{l_1 l_2}, \quad (12)$$

where $y_i = m_i/m_j$ and

$$F(x, y) = \lambda^{1/2}(1, x^2, y^2)[(1 + x^2)(1 + x^2 - y^2) - 4x^2], \quad (13)$$

$$H(x, y) = 12 \int_{z_1}^{z_2} \frac{dz}{z} (z - y^2)(1 + x^2 - z) \lambda^{1/2}(1, z, x^2) \lambda^{1/2}(0, y^2, z). \quad (14)$$

The integration limits are $z_1 = y_{l_2}^2$, $z_2 = (1 - y_{l_1})^2$ and $F(0, 0) = H(0, 0) = 1$. Summing up all the decay modes in (10) one gets for the total ν_j width

$$\begin{aligned} \Gamma_{\nu j} &= 2|U_{\mu j}|^2(\Gamma_2^{(\mu)} + \Gamma_3^{(\mu e)} + \Gamma_3^{(\mu \mu)}) + 2|U_{ej}|^2(\Gamma_2^{(e)} + \Gamma_3^{(ee)} + \Gamma_3^{(e\mu)}) \equiv \\ &\equiv |U_{\mu j}|^2\Gamma_{\nu}^{(\mu)} + |U_{ej}|^2\Gamma_{\nu}^{(e)}. \end{aligned} \quad (15)$$

In the resonant domain (9) $\Gamma_{\nu j}$ reaches its maximum value at $m_j = \sqrt{s_1^+}$. Assuming for the moment $|U_{\mu j}| = |U_{ej}| = 1$, we estimate this maximum value to be $\Gamma_{\nu j} \approx 4.7 \times 10^{-10}$ MeV. Since $\Gamma_{\nu j}$ is so small in the resonant domain (9) the neutrino propagator in the first term of Eq. (1) has a very sharp maximum at $s = m_j^2$. The second term, being finite in the limit $\Gamma_{\nu j} = 0$, can be neglected in the considered case. Thus, with a good precision we obtain from Eq. (1)

$$\Gamma^{res}(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \approx c\pi G(z_0) \frac{m_j |U_{\mu j}|^4}{|U_{\mu j}|^2 \Gamma_{\nu}^{(\mu)} + |U_{ej}|^2 \Gamma_{\nu}^{(e)}} \quad (16)$$

with $z_0 = (m_j/m_K)^2$. This equation allows one to derive, from the experimental bound of Eq. (8), the constraints on ν_j neutrino mass m_j and the mixing matrix elements $U_{\mu j}, U_{ej}$ in a form of a 3-dimensional exclusion plot. However one may reasonably assume that $|U_{\mu j}| \sim |U_{ej}|$. Then from the experimental bound (8) we derive a 2-dimensional $m_j - |U_{\mu j}|^2$ exclusion plot given in Fig. 2. For comparison we also present in Fig. 2 the existing bounds taken from [17]. As shown in the figure, the experimental data on the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay exclude a region unrestricted by the other experiments. The constraints can be summarized as

$$|U_{\mu j}|^2 \leq (5.6 \pm 1) \times 10^{-9} \quad \text{for} \quad 245 \text{ MeV} \leq m_j \leq 385 \text{ MeV}, \quad (17)$$

The best limit $|U_{\mu j}|^2 \leq 4.6 \times 10^{-9}$ is achieved at $m_j \approx 300$ MeV. Note that these limits are compatible with our assumption that $|U_{\mu j}| \sim |U_{ej}|$ since in this mass domain, typically $|U_{ej}|^2 \leq 10^{-9}$ [18].

The constraints from $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ in Fig. 2 and Eq. (17) can be significantly improved in the near future by the experiments E949 at BNL and E950 at FNAL [19]. It is important to notice that in the resonant domain we have

$\Gamma^{res}(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \sim |U|^2$, while outside $\Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \sim |U|^4$. Thus in the resonant mass domain the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay has a significantly better sensitivity to the neutrino mixing matrix element. In forthcoming experiments the upper bound on the ratio in Eq. (8) can be improved by two orders of magnitude or even more. Then this experimental bound could be translated to the limit $|U_{\mu j}|^2 < 10^{-11}$ and stronger.

Finally, it is important to mention the possible cosmological and astrophysical consequences of the hundred-MeV neutrinos considered in the present paper. Massive neutrinos contribute to the mass density of the universe, participate in cosmic structure formation, big-bang nucleosynthesis, supernova explosions, imprint themselves in the cosmic microwave background etc. (see [20] for a review). This implies certain constraints on the neutrino masses and mixings. Currently, for massive neutrinos in the mass region of Eq. (9), the only available cosmological constraints arise from the mass density of the universe, $\tau_{\nu_j} < (\sim 10^{14})$ sec, and cosmic structure formation, $\tau_{\nu_j} < (\sim 10^7)$ sec, (taken from ref. [20]) where τ_{ν_j} is the lifetime of massive neutrinos. On the other hand, on the basis of the formula (15), assuming $|U_{\mu j}|^2 \sim |U_{ej}|^2 \leq 4.6 \times 10^{-9}$ as in Eq. (17), we find conservatively 10^{-2} sec $< \tau_{\nu_j}$. Thus massive neutrinos with masses in the interval (9) do not contradict the known cosmological constraints and there remains a wide open interval of allowed mixing matrix elements: $(\sim 10^{-18}) < |U_{\mu j}|^2, |U_{ej}|^2 < (\sim 10^{-9})$. Big-bang nucleosynthesis and the SN 1987A neutrino signal may lead to much more restrictive constraints. Unfortunately, as yet the analysis [21] of these constraints does not involve the mass region (9). It may happen that these constraints, in combination with our constraints in Eq. (17), close the window for neutrinos with masses in the interval (9). Then the only physics left to be studied using the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ searches would be physics beyond the SM other than neutrino issues. Nevertheless, significant model dependence of all the cosmological constraints should be carefully considered before such a determining conclusion is finally drawn.

4 Conclusion

We studied the potential of the K-meson neutrinoless double muon decay as a probe of the Majorana neutrino masses and mixings. We found that this process is very sensitive to the hundred-MeV neutrinos ν_j in the resonant mass range (9). We analyzed the contribution of these neutrinos to the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay rate and derived stringent upper limits on the Majorana neutrino mixing ma-

trix element $|U_{\mu j}|^2$ from the current experimental data. In Fig. 2 we presented these limits in the form of a 2-dimensional exclusion plot, and compared them with existing limits. The $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay excludes a domain previously unrestricted experimentally. We point out that the current and near future experimental searches for this decay are not able to provide any information on the light eV-mass ($m_\nu \sim \text{eV}$) or heavy GeV-mass ($m_\nu \sim \text{GeV}$) neutrinos, since in those cases the required experimental sensitivities are by many orders of magnitude far from the realistic ones. Finally, we notice that the decay $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ can in principle probe lepton number violating interactions beyond the standard model. A well known example is given by R-parity violating interactions in supersymmetric models. However these aspects of the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay are yet to be studied.

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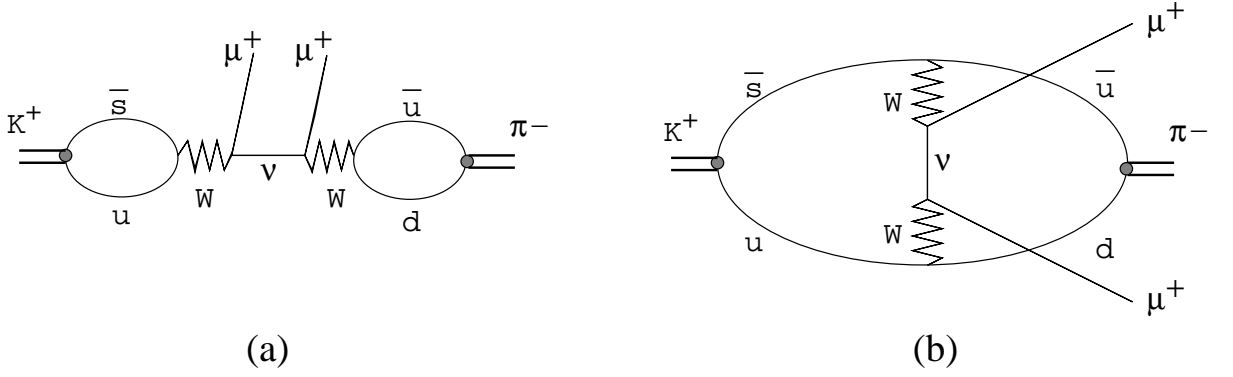


Figure 1: The lowest order diagrams contributing to $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay.

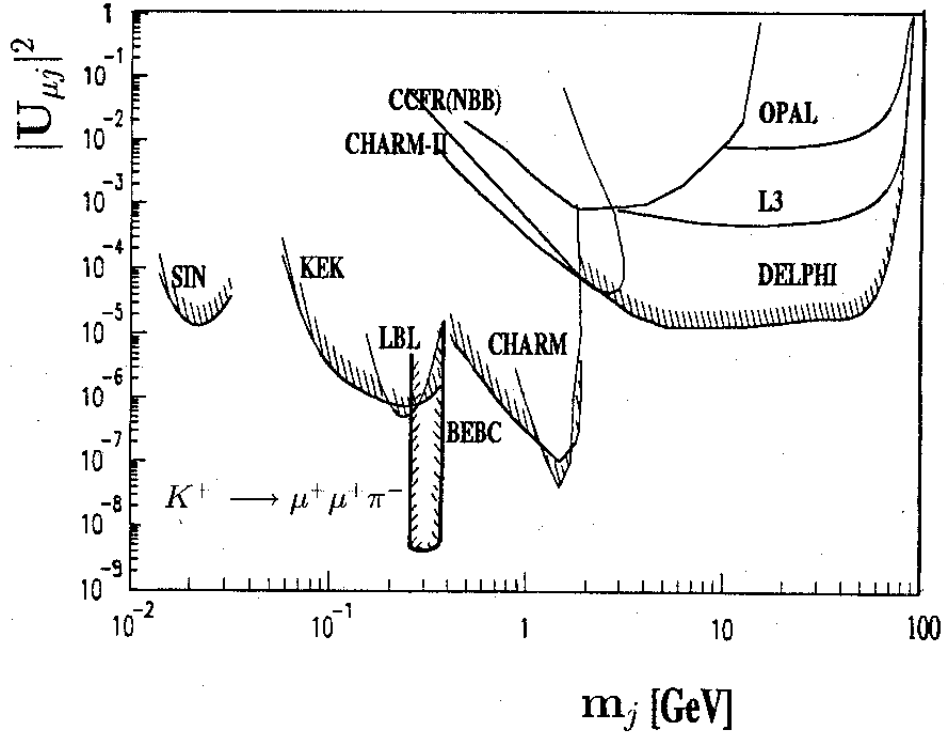


Figure 2: Exclusion plots in the plane $|U_{\mu j}|^2 - m_j$. Here $U_{\mu j}$ and m_j are the heavy neutrino ν_j mixing matrix element to ν_μ and its mass respectively. Domains above the curves are excluded by various experiments according to the recent update in Ref. [17]. Region excluded by $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay [present result] covers the interval $249\text{MeV} \leq m_j \leq 385\text{MeV}$ and extends down to $|U_{\mu j}|^2 \leq 4.6 \times 10^{-9}$.